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HOW TO MAKE A COURSE OF STUDY IN ARITHMETIC

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INTRODUCTION

Purpose of circular. The purpose of this circular is to indicate the application to arithmetic of the general technique of course-of-study making.[1] The circular is intended to furnish a working basis and a guide to those who are making or revising the course of study in arithmetic.[2]

Plan of circular. In order to provide a general view of the content and organization of a course of study in arithmetic, the outline for an arithmetic course of study is given first. The different divisions of this outline are then considered in detail; the nature of the content for each division is discussed, and suggestions are made regarding the procedure to be followed. The appendix includes three types of material: first, the fourth-grade portion of a course of study in arithmetic, which was prepared to illustrate the application of the principles outlined in this circular; second, standards of attainment as derived from standardized tests in the fundamental operations of arithmetic; and third, selected and annotated bibliography, which contains references valuable to those formulating courses of study in arithmetic.

Course-of-study making a cooperative enterprise. The making of courses of study is generally undertaken as a cooperative enterprise. When several or all of the courses of study for a school system are being formulated, the teachers are usually organized into subject committees. In addition there are one or two general committees to oversee the whole work of course-of-study making. They give directions and inspiration to the subject committees and also perform some of the general work, such as specifying the grade time allotments for the various subjects. The subject committees prepare the

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[1]The general principles for making a course of study are given in a previous circular:

Monroe, Walter S. "Making a course of study." University of Illinois Bulletin Vol. 23, No. 2. Bureau of Educational Research Circular No. 35. Urbana: University of Illinois, 1925. 36 p.

[2]See *Ibid.*, p. 3-4, for the distinction between curriculum and course of study, a distinction which should be kept clear at all times.

courses of study for the several school subjects. This circular describes in detail the task of the subject committee on arithmetic. Many superintendents state that this cooperative plan of course-of-study making has proved the most valuable work they have undertaken for the improvement of teachers in service.[3]

[3]For a more detailed discussion of organizing for course of study making and the benefits of such concerted effort, see: Monroe, *op. cit.*

THE GENERAL OUTLINE OF AN ARITHMETIC COURSE OF STUDY

The course of study in arithmetic should contain two types of material: first, specifications, which include objectives (general and specific), organization of topics, and time allotments; second, directions relative to instruction, which include suggestions as to learning exercises and methods of teaching.

Outline for a course of study in arithmetic. The following outline is suggested as a working basis for an arithmetic course of study covering the work of six grades:

I. Introduction
   1. Purpose of this course of study
   2. The general objectives of arithmetic
   3. General organization of the course in arithmetic
      A. Grade time allotments
      B. The sequence of specific objectives from grade to grade (Presented in tabular form)
      C. The sequence of topics from grade to grade (Presented in tabular form)[4]

II. Course of Study by Grades
   1. First grade
      A. Specific objectives
      B. Suggestions relative to instruction
         a. Learning exercises
         b. Methods
   2. Second grade (subdivisions as for first grade)
   3. Third grade (subdivisions as for first grade)
   4. Fourth grade
      A. Specifications
         a. Specific objectives
         b. Topics
            (1) Organization of topics
            (2) Suggested time allotments for topics

[4]This has been effectively done in the elementary course of study for Baltimore. "Arithmetic—course of study for grades four, five, and six." City of Baltimore, Maryland: Department of Education, 1924. "Tabulation of the course of study in arithmetic. Kindergarten, primary and intermediate grades" (inserted inside the back cover).

B. Directions relative to instruction
   a. Learning exercises
   b. Methods
5. Fifth grade (subdivisions as for fourth grade)
6. Sixth grade[5] (subdivisions as for fourth grade)

III. References for the Teacher

It should be noted that in this outline there are essentially three organizations. First, there is the general organization of the course of study as a whole, containing three divisions: introduction, course of study by grades, and references for the teacher (indicated by Roman numerals I, II, and III). The second organization is the logical arrangement of the course in arithmetic, including grade time allotments, specific objectives and topics (placed in Introduction). Finally, the content of the course of study within each grade is organized into a form that is usable by the teacher, giving first the grade specifications and then the directions for carrying them out.

[5]Because of the general development of the junior-high-school movement, this outline ends with the sixth grade. If arithmetic is continued through the seventh and eighth grades, the outline may be continued in its present form. If a "unified mathematics" course is offered above the sixth grade, a somewhat different treatment may be needed.

I. THE INTRODUCTION TO AN ARITHMETIC COURSE OF STUDY

Necessity for a point of view. The introductory section of a course of study should present in definite terms the point of view which is exemplified in the later sections. In other words, the introduction should present briefly the author's educational philosophy as applied to arithmetic. This will include some such topics as the following: the purpose of the course of study, the nature of the learning and teaching processes in arithmetic, the use of the textbook in arithmetic, and the purposes of teaching arithmetic. The last topic leads directly into the discussion of general objectives.

General objectives. The principal function of the discussion of general objectives of arithmetic is to furnish the background for the consideration of the detailed objectives. The following statements are typical of those used in expressing general objectives in arithmetic.[6]

1. To give arithmetical knowledge that fits into "real situations" in the school, home, shop, or social life of the pupils and to prepare them to meet similar situations in adult life.
2. To give power to use mental calculations in every-day demands.
3. To train in proper habits of accuracy, speed, neatness, and checking results in mathematical problems.
4. To give information about the conduct of every-day business.
5. To develop habits of logical reasoning from conditions to results.

Formulating the point of view and general objectives. No doubt anyone who starts to write a course of study in arithmetic will have already an educational philosophy and a set of general objectives, but neither may be clearly formulated. In such instances, it would be well to read one or two books on educational theory and methods of teaching arithmetic before attempting to write out a point of view and general objectives.[7] With a general educational

[6] "Arithmetic—elementary course of study." Trenton: New Jersey: Board of Education, 1923, p. 9

[7] See bibliography for suggested references.

philosophy toward arithmetic fairly well established, the writers of a course of study are ready to proceed with the other tasks indicated in the outline on pages 5-6. They may find later that their point of view has changed in certain details and that they wish to rewrite portions of what they have formulated, but they need these first statements to guide them as they develop the course of study.

Organization of the course in arithmetic. "The organization of the course in arithmetic" should be prepared after the point of view and general objectives have been formulated and before the directions relative to instruction in the separate grades are written. This organization consists of three things: first, the grade time allotments; second, the sequence of the specific objectives from grade to grade; and third, the sequence of topics from grade to grade.

Grade time allotments. The grade time allotments for arithmetic probably will have been determined in advance by a general committee[8] and will need only to be accepted and adjusted harmoniously in the course of study. Time allotments have not been scientifically determined, but the usual practices have been made the subject of study. In an investigation[9] made in forty-nine cities, the following grade time allotments to arithmetic in terms of minutes per week are given:

	Grades							
	I	II	III	IV	V	VI	VII	VIII
Minutes per week.....	96	143	193	206	211	211	212	211
No. of cities giving arithmetic.....	32	48	48	48	48	48	42	36

This table should be read as follows: thirty-two cities, out of forty-nine studied, teach arithmetic in Grade I on an average of ninety-six minutes per week, and so forth. The average time devoted to arithmetic in the forty-nine cities is 1,451 minutes per week.

Sequence of specific objectives and topics from grade to grade.[10] The course in arithmetic should not be considered in a piecemeal fashion, for instance, for one grade in isolation from other grades; but it should be taken as a whole for the elementary school. In order

[8] See page 3 for cooperative organization for course-of-study making.

[9] "Facts on the public school curriculum." Research Bulletin of the National Education Association, Vol. I, No. 5. Washington: Research Division of the National Education Association, 1923, p. 326-27.

[10] Specific objectives and topics are discussed at length later, p. 10-14. The purpose here is only to point out the manner of presenting them in the introductory section of the course of study.

to provide this unified view of the course, the sequence of specific objectives and of topics from grade to grade should be presented in the introductory section of the course of study. Probably such a presentation can be made most effective when given in tabular form.[11] An illustration of the beginning of such a form is given below:

SEQUENCE OF TOPICS IN ELEMENTARY ARITHMETIC .

Grade	Counting	Reading and Writing Number		Etc.
		Arabic	Roman	
I				
II				
III				
Etc.				

This same form should be used for the specific objectives so that they also may be presented as a unified whole.

Both of these organizations form a summary of the specifications as formulated for the various grades. Although they are given in the introductory section of the course of study, they cannot be prepared until the specifications for each grade have been formulated.

[11]A good example of this may be found in the elementary course of study for Denver, Colorado:

"Arithmetic—grades 1, 2, 3, 4, 5, and 6—elementary school," Denver: Board of Education, 1924, (insert inside front cover).

II. COURSE OF STUDY BY GRADES

General content of course of study by grades. As was noted, page 5, the arithmetic course of study by grades should contain two types of material: first, specifications; second, directions relative to instruction. The specifications include time allotments, specific objectives, and topics, and should be presented at the beginning of the portion of the course of study for each grade. The suggestions on how to direct the pupils in their learning so as to accomplish the objectives should be very specific and should pertain directly to the specifications for each grade. The outlines for the first three grades, as given on page 5, are less elaborate than those for the later ones, because of the informality of the work in the lower grades.[12]

General nature of specifications. The grade specifications include specific objectives, organization of topics, and time allotments. The last needs no discussion here because it is treated in detail on page 8.

Nature of specific objectives. Specific objectives, as the name implies, are particularized statements of general objectives. They should be as detailed and definite as possible and should be expressed in terms of ability to do. Whenever feasible, they should specify the degrees of the abilities to be attained. For example, a specific objective in both fifth and sixth-grade arithmetic might be "the ability to multiply four-place numbers by two-place numbers," but this statement is still of too general a nature to differentiate between the fifth and sixth-grade standards. The degree of the ability to be attained should be added. Thus, in the fifth grade the pupils should attain "the ability to multiply four-place numbers by two-place numbers at the rate of eight examples in eight minutes," while in the sixth grade the rate should be "nine examples in six minutes."

Whenever standards of this nature have been worked out, they should be used by the course-of-study writers. However, there are many other abilities to be engendered by arithmetic for which degrees of attainment cannot be so definitely stated, as for example, in the following objectives: In Grade I, "the ability to read numbers to

[12]See footnote, page 12, for suggestions relative to the time for beginning formal work in arithmetic.

100;" in Grade II, "the ability to write numbers to 1,000;" in Grade VI, "the ability to originate problems," or "the ability to draw to scale." Even though the degrees of attainment in such abilities have not been worked out, and may never be in all cases, they should be stated in as definite terms as possible.[13] General terms such as "proficiency," "fluently," and "quickly" and statements in terms of ground to be covered or of so many pages in the textbook should be avoided.

How to formulate specific objectives. There are several sources from which specific objectives may be obtained: first, arithmetic textbooks;[14] second, books on methods of teaching arithmetic; third, numerous courses of study; fourth, various kinds of standardized tests in arithmetic; and finally, special studies such as the one made in Iowa by Wilson.[15] The information of those who formulate the objectives may be considered as another source, but their knowledge and judgment should be used more for the purpose of modifying and adjusting the objectives found elsewhere. This function of judgment needs to be performed generously.

After the specific objectives have been collected from as wide a range of sources as possible, they must be culled or selected. A certain amount of rejecting may go on when they are being listed, but in order that they may be seen as a whole and in their proper relations to each other, it is probably better to avoid making the selections until a rather complete list has been made up. At this point, for the first time, the judgment of the curriculum builder enters to an appreciable extent. He should be guided by principles previously decided upon which are flexible and allow for a liberal interpretation and yet definite enough to give assistance. Examples of principles that are really helpful are given in the following statements:[16]

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[13]For illustrations of specific objectives see the appendix: p. 23-26.

[14]In general, textbooks only imply objectives to be attained, although some of the newer arithmetics are including standards of attainment. For example:

Watson, Bruce M. and White, Charles E. Modern Intermediate Arithmetic. Boston: D. C. Heath and Company, 1922. 254 p.

[15]Wilson, Guy M. "A survey of the social and business uses of arithmetic." Sixteenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1917, p. 128-42.

[16]Caldwell, Otis W. "Types and principles of curricular development," Teachers College Record, 24:326-37, September, 1923.

1. Mathematics, in the elementary and junior-high-school grades, should be primarily a tool for the quantitative thinking that children and adults need to do.

2. Each year should give the most intrinsically valuable mathematical information and training which the pupil is capable of receiving at that time, with little consideration of the needs of subsequent courses.

3. This aim necessitates the inclusion in junior-high-school grades of certain elements of arithmetic, intuitive geometry, algebra, trigonometry, and statistics, although these are not to be rigidly classified under the traditional divisions as named.

4. Manipulation of mathematical symbols as an end should be omitted.

5. Attention should be directed toward a better appreciation of the part that mathematics has occupied and is now occupying in the progress of civilization.

6. There should be a marked increase in the accuracy of computation with integers, fractions, and percents.

Organization of specific objectives. The specific objectives of arithmetic need to be organized in proper sequence, so that their realization will mean a gradual yet sure development of arithmetical ability on the part of the pupils. While the objectives are being collected, they will no doubt be assigned tentatively to particular grades; but after a fairly complete list has been made, they will need to be gone over carefully and reorganized so as to assure a unified whole.[17] A few guiding principles, such as the following, should be formulated before making the reorganization:

1. Provisions must be made usually for the development of an ability over a period of more than one year.

2. The introduction of totally new objectives should not be too rapid.

3. The organization should involve a "psychological" rather than a strictly "logical" sequence.

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[17]One of the largest factors in determining the grade placement of objectives is the grade in which formal work in arithmetic is begun. It seems that no formal work should be given in the first grade, probably none in the second and very little in the third. For a detailed discussion see:

Wilson, Guy M. "Arithmetic," Third Yearbook of the Department of Superintendence. Washington: Department of Superintendence of the National Education Association, 1925, p. 37-40.

Presentation of specific objectives. The specific objectives should be presented not only at the beginning of the course of study for each grade, but also in the introductory section of the course of study, as suggested on page 9.

Relation of specific objectives to topics. It is best to complete the organization of the specific objectives before outlining the topics, for theoretically the specific objectives should guide the latter task. However, in planning an outline of topics for a year, some new objectives probably will be suggested. In order to insure that the outline is compatible with the list of objectives, a careful comparison should be made. The writer of a course of study should make certain also that all topics implied in the list of objectives have been included in the outline.

Selection of topics. The sources of suggestions for the topics of arithmetic are much the same as for specific objectives: textbooks, books on methods of teaching arithmetic, courses of study, standardized tests, and special studies.[18] Chief among these are textbooks, courses of study, and special studies. These three sources should be thoroughly canvassed and the topics outlined as much in detail as possible. It is also well to make approximate grade placements before making comparisons with the specific objectives.

Organization of topics. In the organization of topics, the specific objectives and the topics should be compared and additions or subtractions made in accordance with the results of scientific studies, with the judgments of those who are writing the course of study, and with principles previously set up. At the same time, the final organization of the topics and objectives in relation to each other should be made. As any organization that is determined upon must be coordinated with the textbooks to be used, due consideration should be given to the arrangement and development of the topics in the text.

When this organization has been completed, the topics should be arranged in tabular form as suggested on page 9, in order that they may be seen as a whole by each teacher. Such a table presents graphically the relation of the work in each grade to that of all the

[18] For a digest of the special studies that have been made on the arithmetic curriculum see:

Wilson, Guy M. "Arithmetic," Third Yearbook of the Department of Superintendence. Washington: Department of Superintendence of the National Education Association, 1925, p. 35-109.

other grades, and the relation of the specific objectives to the topics. With these two points of view, a teacher's work should be much more effective than if she saw only the work of her grade in isolation.

Provisions for individual differences by modifications of specific objectives and topics.[19] In formulating specific objectives and in selecting topics, consideration should be given to provisions for individual differences. Some eliminations may be made for the slower pupils and some additions for the brighter ones. All such provisions should be carefully considered and not decided upon hastily. If the school is organized into "X, Y, Z" ability groups, greater variations may be made than if there is no segregation. However, most of the provisions for individual differences, especially in the latter instance when homogeneous grouping is not attempted, must be made in adjusting learning exercises and methods of instruction.[20]

General nature of suggestions relative to instruction. Suggestions relative to instruction fall into two classes; those relating to learning exercises, and those to methods of instruction.

Suggestions for learning exercises. Arithmetic textbooks are chiefly compilations of learning exercises, but teachers of arithmetic must devise many additional exercises in order that pupils may achieve the objectives set for them. Especially is this true in teaching and solving problems that involve new words. Additional reading exercises are often needed, explanations by the teacher are necessary, or the pupils may be required to make diagrams or to do many other types of exercises not given in the textbook.

The process of long division may furnish a detailed example of the need for formulating and assigning appropriate learning exercises and of the assistance that may be given the teacher by the course of study. In achieving the objective, "ability to do long-division examples of the type $25\overline{)6775}$ at the rate of four in eight minutes," which is an appropriate objective for a fourth-grade class to achieve at the end of the year, there are a variety of learning exercises that the teacher might set up for the pupils to do. Suitable learning exercises when the process is first being learned are different from appropriate ones when long division is understood as a process but

[19] Two courses of study which make very definite subject-matter provisions for individual differences are the elementary course of study in arithmetic for Long Beach, California, and that for Trenton, New Jersey.

[20] Modifications of learning exercises and methods of instruction are discussed later, pages 15, 20; 34-36.

when the ability to perform the operations with sufficient rapidity has not been attained. In formulating the learning exercises, care must be taken to avoid having the pupil meet too many difficulties at first. Osburn[21] points out that there are four centers of trouble in learning to do long division: "(1) in getting acquainted with the new form, (2) in carrying, (3) in borrowing, and (4) in estimating the quotient including the use of zeros." It is important that the learning exercises provide for the mastery of all these difficulties yet not introduce them all at the beginning. Osburn cites one book that introduces the child to long division through the problem $15\overline{)240}$, in which all the difficulties mentioned above are encountered.

After the pupils understand long division as a process but do not have the ability to do such exercises with sufficient rapidity, drill exercises that are of a different nature from the earlier learning exercises should be given. In order to engender a complete mastery of the process, teachers should assign as learning exercises problems that involve long division but in which the pupil must decide when and when not to use the process. Such problems would be entirely inappropriate until the pupils had a thorough understanding of long division as a process.

In the part of the course of study that deals with the fourth grade, the teacher's attention should be called to the appropriateness and inappropriateness of different types of learning exercises which might be used in teaching long division, and suggestions should be made as to those types of exercises which may be used with the greatest effectiveness. If the textbook happens to be weak in its method of handling long division, the course of study can be especially helpful to the teacher.

It should be borne in mind by those who write the course of study that learning is an active process, that the pupil learns only through his own activity, physical or mental, and that there is no such thing as a pouring-in process or an inscribing on a blank tablet. Effective activity on the part of the pupil must be provided for by the teacher.

Effect of interests and local conditions on learning exercises. In educational writings and discussions, much attention is given to a consideration of local conditions and interests in making curricula and in writing courses of study. There is opportunity in formulating

[21] Osburn, Worth J. Corrective Arithmetic. Boston: Houghton Mifflin Company, 1924, p. 68-69.

learning exercises not only to provide for peculiarities of needs due to local conditions and interests but to take advantage of them. A water reservoir, a dam, or a factory may furnish admirable material for learning exercises that would not be effective where these did not exist. All communities have conditions that are peculiar to them and of which advantage can be taken. It is possible that the same objectives and curriculum will function as well in any one of a large number of communities as in any other, but it is easily conceivable that many significant local differences exist which may be pertinent and valuable in formulating learning exercises.[22]

General nature of suggestions on methods of instruction.[23] The course of study performs a supervisory function. Its purpose is to help and to coordinate the work of the teachers of a school system. Some teachers have a well-developed technique as a result of training and experience; others are relatively inexperienced and have little training; others have not taught in the particular grade in which they are now teaching; and still others are new to the school system, although experienced and well-trained. The course of study should help all these teachers to use appropriate methods in particular grades and with given subject-matter. It should help assure a uniformity of method where such uniformity is desirable. For instance, if the Austrian method of subtraction is used by some teachers, it should be used uniformly throughout the school system. The suggestions on appropriate methods should follow the suggestions on learning exercises, and should include at least the following items: motivation (including games), lesson types, oral and written work, use of textbooks, testing and remedial instruction (including standardized and informal tests and diagnosis), practice tests, adaptation of instruction to individual differences, and supervision of study.

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[22]A word of warning is not amiss here. The adaptation of learning exercises to local conditions should not be carried to an extreme. For a good example of what not to do, see:

Brown, Joseph C. and Coffman, Lotus D. How to Teach Arithmetic. Chicago: Row, Peterson and Company, 1914, p. 72.

[23]For a digest of the results of scientific investigations relative to methods of instruction in arithmetic, see:

Monroe, Walter S. "Principles of method in teaching arithmetic, as derived from scientific investigation." Eighteenth Yearbook of the National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1919, p. 78-95.

Methods of instruction: motivation.[24] The chief assistance on motivation which the course of study in arithmetic can give the teacher is in suggesting suitable types of games, field trips, and supplementary projects. These are learning exercises with a large motivating element. It would not be out of place to discuss games and field trips under learning exercises rather than under motivation, but in either place the suggestions should be of a concrete rather than of a theoretical nature. Specific games and projects should be given. This is another excellent opportunity to take advantage of local conditions.

The course of study should make clear the "motivating elements"[25] that are effective in the different grades. Curiosity, pleasure in successful work, manipulation, and many other bases of motivation are powerful throughout life, but interests as motivating elements change greatly from grade to grade. Means of motivation that are successful in the second grade may not be so in the fourth, fifth and sixth grades. Those motivating factors that should be avoided and those of which most advantage should be taken in particular grades also should be considered. Some teachers have a tendency to "over motivate" or to motivate merely for the sake of motivation. The course of study can assist in overcoming such a habit by pointing out the ill effects of certain practices such as the use of distracting games.

Methods of instruction: lesson types. In arithmetic there are different types of lessons. The three outstanding types are the development lesson,[26] the drill lesson, and the lesson that involves plays and games. The course of study should make a distinction between these three types and may well give an outline of an illustrative lesson for each. The characteristics of the good drill lesson should be pointed out especially, for there has probably been

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[24]The term "motivation" is used here as defined by Lennes (p. 125): "Motivation in elementary work consists in so combining those activities which are desired of the child with other activities in which he is spontaneously and directly interested that the combination of activities will to him be interesting and attractive."

For an excellent discussion of motivation and fruitful suggestions, see:

Lennes, N. J. The Teaching of Arithmetic. New York: The Macmillan Company, 1923, p. 119-67.

[25]The term "motivating elements" is used to include such factors as instincts, acquired interests and many others which are appealed to for motivation.

[26]For illustrative lesson outlines, see:

"Arithmetic—elementary course of study." Trenton, New Jersey: Board of Education, 1923. 96 p.

too much of a tendency to waste time in inefficient drill or to fail to give enough drill.[27] The lesson that involves plays and games will need little more than mentioning at this point in the course of study because of the discussion elsewhere under either motivation or learning exercises.

Methods of instruction: oral and written work. The type, amount, and proportion of oral and written work in arithmetic vary from grade to grade. There are also variations made in the time of doing the written work; that is, in recitation time, in school but outside of recitation time, or at home. For the attainment of some objectives, oral work is the more effective, for others, written work is more successful. Often a proper proportion of each is desirable. Some written work can be done as well at home as at school; some, especially when the pupils are not thoroughly familiar with the process involved, should be done only under the supervision of the teacher. On all such questions, the course of study should give assistance to the teacher.

Methods of instruction: use of textbooks. The textbook in arithmetic is probably more used and more slavishly followed than the text in any other subject. It should be considered a guide and an economical source of organized subject-matter and learning exercises, but it must be kept subordinate to the specific objectives of the curriculum. There is need for omitting some parts as well as for supplementing other parts. The course of study should point out these facts, and especially should show from grade to grade the specific use that should be made of the textbook. Undoubtedly its use varies greatly; in the first grade it probably should not be introduced, later it becomes an important instructional instrument. As progress is made through the grades, the textbook becomes less and less based upon the pupils' experiences and gives more and more information other than arithmetical knowledge.[28]

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[27]For a résumé of the studies which have been made relative to drill in arithmetic, see:

Wilson, Guy M. "Arithmetic." Third Yearbook of the Department of Superintendence. Washington: Department of Superintendence of the National Education Association, 1925, p. 63-91.

[28]For a discussion of the introduction of subject-matter other than that which is purely arithmetical, see:

Lennes, N. J. The Teaching of Arithmetic. New York: The Macmillan Company, 1923, p. 171-90.

Methods of instruction: use of tests.[29] There have been published a large number of standardized tests on the various phases of arithmetic. They may be used by the classroom teacher for the following purposes: (1) promotion and classification of pupils, (2) diagnosis of pupils' difficulties in order to provide remedial instruction, and (3) evaluation of teaching efficiency. The course of study should not attempt to be a treatise on the use of standardized tests, but should instruct the teacher regarding available tests, occasions for their administration, and the uses to be made of the results.[30] From the standpoint of the teacher, remedial instruction is the most important use that can be made of the results of tests. The course of study should point out the types of difficulties that are met most often and some of the means of overcoming them.[31] The need for a great deal of testing besides that done by standardized tests should be pointed out also. Types of tests, such as the traditional and the new examination, and the uses to be made of the results should be discussed.[32]

Methods of instruction: practice tests.[33] A number of lists of exercises, usually called practice tests, have been carefully planned so as to provide the drill needed to engender certain specified degrees of ability. If such practice tests are considered desirable for the

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[29]For a list of available tests and a discussion of their purposes, see:

Doherty, Margaret and MacLatchy, Josephine. "Bibliography of educational and psychological tests and measurements." U. S. Bureau of Education Bulletin, 1923, No. 55. Washington: Government Printing Office, 1924. 233 p.

Odell, Charles W. "Educational tests for use in elementary schools, revised." University of Illinois Bulletin, Vol. 22, No. 16. Bureau of Educational Research Circular No. 33. Urbana: University of Illinois, 1924. 22 p.

[30]For an adequate discussion of testing in arithmetic, see:

Monroe, Walter Scott, DeVoss, James Clarence, and Kelly, Frederick James. Educational Tests and Measurements. (Revised and Enlarged.) Boston: Houghton Mifflin Company, 1924, p. 19-93.

[31]The most recent and probably the best discussion of remedial instruction in arithmetic is:

Osburn, Worth J. Corrective Arithmetic. Boston: Houghton Mifflin Company, 1924. 182 p.

[32]An excellent discussion of written examinations and the technique of informal testing is found in:

Monroe, Walter S. and Souders, Lloyd B. "The present status of written examinations and suggestions for their improvement." University of Illinois Bulletin, Vol. 21, No. 13. Bureau of Educational Research Bulletin No. 17. Urbana: University of Illinois, 1923. 77 p.

[33]The bibliographies just referred to under the use of tests include practice tests.

particular school system, the course of study should specify those to be selected and make provisions for their systematic use. No elaborate discussion is necessary since ample directions are provided by the publishers, but the teachers should familiarize themselves with these instructions.

Methods of instruction: adaptation of instruction to individual differences.[34] There are four outstanding types of provisions for individual differences which may be made: first, modifications of objectives; second, modifications of topics; third, variations in types of learning exercises; and, fourth, modifications of methods of instruction. The modifications of objectives and topics should be indicated in the first outlines of specifications given in the introductory section of the course of study and should be repeated when the specifications are made for each grade. Variations in learning exercises and modifications of methods of instruction should be discussed in the portions of the course of study dealing with the particular grades. It is more effective to group these suggestions under each grade division than to separate them into distinct topics.

Methods of instruction: directing study. The most important function of the teacher is to direct the learner in his doing of learning exercises. Much of the activity of the learner must be carried on in so-called study periods, whether these be a portion of the recitation period or entirely separate. The teacher's success depends mainly upon her ability to direct the pupils in their study. She must be able to give effective directions for work, to know when to give assistance, and to decide the kind of assistance which will be most helpful. The course of study cannot give the teacher all of this ability, but it can suggest the type of activity on her part which is most likely to be effective. Any discussion of supervised study is so closely related to all other phases of instruction that ordinarily there should be a general permeation of it throughout other topics, such as the use of textbooks, oral and written work, and the use of practice tests. If in a school system there is a special scheme of supervised study, such as the Batavia Plan, there is need to give specific directions regarding the particular form of supervised study so that the instruction in arithmetic may fit in with the general scheme.

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[34]One of the better courses of study in its provisions for individual differences is the Long Beach, California, course of study in arithmetic.

Methods of instruction: summary. Although many topics (motivation, lesson types, oral and written work, use of textbooks, testing and remedial instruction, practice tests, adaptation of instruction to individual differences and supervision of study) have been discussed under the general subject of methods of instruction, all of these should not be treated in great detail in the outlines for each grade. Some of the topics will need to be dealt with much more extensively in certain grades than in others. The manner of presenting these various topics should be to fuse them together rather than to force too many arbitrary lines of distinction. For instance, the treatment of lesson types and of oral and written work readily fuse into each other, yet there need be no confusion about either. If the reader will again refer to the outline of a course of study in arithmetic, pages 5-6, this fact may be somewhat more evident.

APPENDIX

COURSE OF STUDY IN ARITHMETIC GRADE IV[35]

The textbook. The textbook to be used is Drushel, Noonan, and Withers, Arithmetic Essentials—Book One,[36] p. 161-304.

Practice tests. This course of study does not require the use of practice tests, but, if they are desired by the school, it makes provisions for the Courtis Standard Practice Tests.[37]

Standardized achievement tests. It is not necessary that standardized achievement tests be used, as standards of achievement are given later under "objectives and standards of attainment." However, if the use of such tests is considered desirable, provision is made for the following: Monroe's General Survey Scales in Arithmetic, Scale I, forms 1 and 2; Monroe's Diagnostic Tests in Arithmetic, Parts I and II; and Monroe's Standardized Reasoning Tests in Arithmetic.[38]

I. SPECIFICATIONS

Objectives and standards of attainment. The following objectives should be attained before pupils are promoted to the fifth grade. In some cases the degree of attainment in each ability is not stated and must be left to the teacher's judgment. But for the fundamental operations of addition, subtraction, multiplication, and division, the degrees of attainment are stated and should be adhered to strictly.

The time allowances in parentheses are intended for the slower pupils, who may be promoted if they can perform the operations accurately within the somewhat longer time limits. Normal and bright pupils should be held to the shorter time limits. They should

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[35]As an example of an arithmetic course of study constructed in accordance with the principles set forth in this circular, the writer has prepared this portion for the fourth grade. It is not intended to be perfect, but it does furnish a fair illustration of what such a course of study is like. It is assumed that there are similar sections for preceding and succeeding grades, and an introductory section as described on pages 7-9 of this circular.

[36]Drushel, J. Andrew, Noonan, Margaret E., and Withers, John W. Arithmetic Essentials—Book One. Chicago: Lyons and Carnahan, 1921. 304 p.

[37]Published by the World Book Company, Yonkers-on-Hudson, New York, and Chicago, Illinois.

[38]These tests are published by the Public School Publishing Company, Bloomington, Illinois.

not be drilled to exceed these degrees of attainment, but they should be excused from further drill when they have reached these standards and for so long a time as they maintain them.

Those objectives that are starred (*) may be omitted for the slower pupils without hampering them materially in future grades and need not be held as prerequisites for promotion.

The teacher should see the objectives for this grade in their relation to all the objectives for elementary-school arithmetic. A tabulation of these is presented in the introductory section.

Integers:

1. Ability to read numbers as large as 999,999,999.
2. Ability to write numbers as large as 999,999,999.

*3. Ability to read Roman numerals through C, and to know the value of D and M.

*4. Ability to write Roman numerals through C, also using D and M.

*5. Ability to tell time on a clock which has Roman numerals.

*6. Ability to tell the days of the week and month on a calendar.

*7. Ability to read a Fahrenheit thermometer.

8. Ability to recall the multiplication tables of 10's, 11's, and 12's through 12.

9. Ability to do the following types of addition examples at the following rates:

Example	*Number*	*Time*	*Example*	*Number*	*Time*
4			7		
7	17	1 min.	6		
2		(1½ ")	6		
			5		
927			0		
379			5	4	4 min.
756			1		(6 ")
837	6	8 min.	8		
924		(12 ")	7		
110			3		
854			3		
965			1		
344			2		

10. Ability to do the following types of subtraction examples at the following rates:

Example	*Number*	*Time*	*Example*	*Number*	*Time*
107795491	7	4 min.	37	6	1 min.
77197029		(6 ")	5		(1½ ")
			739	3	1 min.
			367		(1½ ")

11. Ability to do the following types of multiplication examples at the following rates:

Example	*Number*	*Time*	*Example*	*Number*	*Time*
6572	3	1 min.	8246	6	6 min.
6		(1½ ")	29		(9 ")

12. Ability to do the following types of division examples at the following rates:

Example	*Number*	*Time*	*Example*	*Number*	*Time*
$8\overline{)3840}$	1	1 min. (1½ ")	$25\overline{)6775}$	4	8 min. (12 ")

Common fractions:

1. Ability to find 1/6 and 1/8 of single things.

2. Ability to find 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6, 1/8, 3/8, 5/8, 1/9, 1/10, 3/10, 7/10, and 9/10 of groups of things. (Without remainders for slower pupils.)

3. Ability to find 1/11 of multiples of 11 through 132.

4. Ability to find 1/12 of multiples of 12 through 144.

5. Ability to do the following types of addition and substraction examples with fractions at the following rates:

Example	*Number*	*Time*	*Example*	*Number*	*Time*
3/5+1/5=	14	1 min. (1½ ")	6/9−4/9=	14	1 min. (1½ ")

Decimals:

1. Ability to read U.S. money expressed decimally as large as $999,999,999.99.

2. Ability to write U. S. money expressed decimally as large as $999,999,999.99.

3. Ability to multiply U. S. money expressed decimally as large as $999,999.99 by a two-place multiplier.

4. Ability to divide U. S. money expressed decimally as large as $999,999.99 by a two-place divisor.

Denominate numbers:

1. Ability to recall the following tables as given in their limited form on page 304 of the textbook:

Money	Length	Area	Liquid measure
Time	Weight		Dry measure

Practical measurements:

1. Ability to make change with amounts not larger than one dollar.

2. Ability to measure accessible rectangular surfaces within limits of one acre.

3. Ability to estimate reasonably well rectangular surfaces within limits of one square rod.

4. Ability to weigh objects within limits of one hundred pounds.

5. Ability to estimate reasonably well the weight of objects within limits of fifty pounds.

6. Ability to measure liquids within limits of five gallons.

7. Ability to estimate reasonably well quantity of liquids within limits of ten gallons.

8. Ability to measure grains and the like within limits of one bushel.

9. Ability to estimate reasonably well quantity of grains and the like within limits of five bushels.

Problems:

1. Ability to solve one-step problems that involve the subject-matter (denominate numbers, decimals, etc.) of this grade. For example:

A boy mows lawns for twenty-five cents an hour. It took him five hours to mow a large lawn. How much did he make?

2. Ability to solve two-step problems that involve making change within limits of $100.00. For example:

A boy bought a pencil tablet for ten cents and a pencil for three cents. He gave the clerk twenty-five cents. How much change should he receive?

NOTE—Two-step problems, other than those involving the making of change, and problems involving more than two steps in making change, should not be attempted in this grade.

Nomenclature and symbols:

1. Ability to use the following terms:

A. M.	multiplicand	plus
P. M.	multiplier	less
addend	product	multiplied by
sum	long division	divided by
minuend	dividend	integer
subtrahend	divisor	common fraction
difference	quotient	Arabic numeral
	remainder	Roman numeral

Generalized habits: (To be striven for in all grades, but especially emphasized in the fourth.)

1. Habit of checking answers.
2. Habit of thinking through the solution of a problem before doing the computation.
3. Habit of working with initiative and independence.

Attitudes: (To be striven for in all grades, but especially emphasized in the fourth.)

1. Pride in one's ability to use numbers.
2. Pride in one's ability to attain standard achievement.

Outline of subject-matter. The subject-matter outlined below is essentially that presented in the textbook but is presented here in a *logical* outline in order that the teacher may see the work of this grade as a whole and in proper relation to the objectives to be attained. She should also see the work of this grade in its proper relation to the entire arithmetic curriculum. This is presented in tabular form in the introductory section.

The subject-matter that is starred (*) is not essential to future progress and may be omitted by the slower pupils. There are relatively few subject-matter provisions for individual differences in this grade. Since arithmetic is a tool subject, there should be comparatively little difference in the subject-matter of this grade for pupils of various abilities. The chief provisions should be excusing from additional work when standards of attainment and objectives have been achieved and making modifications of method. These will be discussed in a later section on methods of teaching, pages 32-35.

Integers:

1. Arabic notation and numeration through hundred millions.
*2. Roman numbers through C. Also D and M.
3. Addition.
4. Subtraction.
5. Multiplication—tables of 10's, 11's, and 12's through 12.
6. Short division.
7. Long division with one and two-place divisors and three and four-place dividends.

Common fractions:

1. 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, 5/6, 1/8, 3/8, 5/8, 1/9, 1/10, 3/10, 7/10, 9/10.
2. Addition of fractions of same denominator.
3. Subtraction of fractions of same denominator.

Decimals:

1. U. S. money to $999,999,999.99.
2. Multiplication of U. S. money as large as $999,999.99 by one and two-place multipliers.
3. Division of U. S. money as large as $999,999.99 by one and two-place divisors.

Denominate numbers:

1. Time:
   Second, minute, hour, day, week, year, leap year, century.
2. Money:
   Cent, dime, quarter, half dollar, dollar.
3. Length:
   Inch, foot, yard, rod, mile.
4. Weight:
   Ounce, pound, ton.
5. Area.
   Square inch, square foot, square yard, square rod.
6. Liquid measure:
   Pint, quart, gallon.
7. Dry measure:
   Quart, peck, bushel.

II. DIRECTIONS RELATIVE TO INSTRUCTION

1. Learning Exercises

Textbook largely composed of learning exercises. The arithmetic textbook is composed chiefly of learning exercises: problems, examples, drill exercises, and so forth. The task of the arithmetic teacher with respect to learning exercises is light as compared with that of teachers of other subjects, such as history where comparatively few such exercises are given in the text. The principal duties of the arithmetic teacher in this respect are first, to select the appropriate exercises from those given in the textbook; and second, to find or formulate supplementary learning exercises when those given are not sufficient to achieve a given objective, when they are inappropriate, when the teacher sees an opportunity to take advantage of her pupils' interest or of peculiar local conditions, or when some other need or opportunity arises.

Additional learning exercises needed. There are a few places in Drushel, Noonan, and Withers' book which are particularly weak, and which should be supplemented either by the use of supple-

mentary textbooks or by exercises devised by the teacher. One such place is on page 166-67 where the topic of Roman numerals is taken up. Those pupils who are taught to read, write, and use Roman numerals must be given more practice than is provided here if they are to attain any real proficiency. The topic should be developed much more gradually. The teacher should have the pupils write numbers in succession from 1 to 100, giving both the Arabic and the Roman. A bird's eye view of all of the letters used may be given at first, but no emphasis should be placed upon the individual letters until they are needed. Thus, the meaning of L should not be stressed until the pupils are ready to write fifty. After the pupils are able to write to XII, they may be taught to tell time on a watch or clock which has Roman numerals. They should also read the numbers of chapters in books, the numbers of volumes of papers and magazines; and if interest is sufficiently aroused, they may learn to read dates on corner stones of buildings.

There is little reason "for teaching any Roman numerals except those from one to twenty, fifty, one hundred, five hundred, and one thousand;"[39] and the more gifted pupils are probably the only ones who should be taught these. But Roman numerals, when taught at all, should be so thoroughly taught that there is real mastery. "Smattering" is not justifiable. Frequent reviews and repetitions throughout the year will be necessary.

The objective of reading and writing Arabic numbers as large as 999,999,999 is not provided for in the exercises of the text. The work on page 189 should be extended to include such numbers. Pupils of the fourth grade usually like to read, write, and "say" large numbers. Little contests similar to "spell-downs" can be used to help teach these large numbers, and newspaper articles containing large numbers can be brought for the pupils to read. In all teaching of numbers there should be a minimum amount of attention, probably none, given to the reading of units, tens, hundreds, thousands, and so forth; but rather the pupils should be taught at first merely to read the numbers as they are ordinarily read: 54 (fifty-four), 123 (one hundred twenty-three), and so forth. "Beginning with the thousands pupils should be taught to read by the 'period method,'

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[39]Brown, Joseph C. and Coffman, Lotus D. How to Teach Arithmetic. Chicago: Row, Peterson and Company, 1914, p. 147.

designating each group of three number places as shown in the following scheme:

billions	*millions*	*thousands*	*ones*
000	000	000	000
		12	638
	146	921	345
9	000	000	000

"A number in any period is read as is any one, two, or three place number and is followed by the name of the period in which it is placed. The name of the 'ones' period is always omitted in reading numbers. Pupils should be trained to avoid the use of 'and' in reading whole numbers."[40]

The teaching of long division is one of the most difficult tasks of the fourth grade. The learning exercises provided in the textbook are adequate and well-graded for most children, so that there will be relatively little need to devise supplementary ones. But in order to teach most effectively, the teacher should be sure that she thoroughly understands the difficulties to be met and the types of exercises that are best suited to meet these difficulties. Especially should care be taken that the pupils do not establish bad habits of work or inhibitions due to beginning with exercises that involve too many or too great difficulties or to moving forward too rapidly. Osburn[41] in his Corrective Arithmetic analyzes the situation well. Space does not permit a repetition here, but the teacher should familiarize herself with this discussion.

With the exception of long division, the teaching of fractions is the most difficult topic in the fourth grade. Here again Osburn's book offers some assistance. The teacher should read pages 47, 50, 52, and 54. The work in fractions is left until the end of the year and is so gradually and thoroughly developed that there is no need for the teacher to devise additional learning exercises for most pupils.

Drushel, Noonan, and Withers' book has a great variety of learning exercises which in their general way are closely tied up with the everyday life of the pupils. But often they lack point because they are phrased in broad terms such as "Some boys in the fourth

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[40]"Arithmetic—course of study for grades four, five, and six." Baltimore, Maryland: Department of Education, 1924, p. 24-25.

[41]Osburn, Worth J. Corrective Arithmetic. Boston: Houghton Mifflin Company, 1924, p. 68-70, 173-77.

grade."[42] Such exercises are very suggestive and can often be changed by the teacher or used as models for other exercises that will be even more "alive" to the pupils. Whenever making such changes or additions, the teacher should remember that these exercises have been carefully arranged and graded so as to introduce the pupils to new processes gradually and to review thoroughly old processes. Care should be taken not to violate these precautions. One of the greatest dangers in so-called "practical" problems is the too sudden or too rapid introduction of new difficulties. Another danger is the entire neglect of some phases of processes which must be given attention if the processes are to be fully mastered. If the teacher avoids these two dangers and knows what purposes she means to accomplish by exercises that she may devise or have the pupils devise, she will often accomplish results which could not be achieved by using the book exercises alone.

Some additional exercises that may be worked out, often in the form of projects by the brighter pupils, are the following:

A live map.[43]
Making a toy shop.[44]
How I may save and help my parents to save.
Operating a ticket office.
Keeping a general merchandise store.
Raising money by selling garden produce.
Making money by raising rabbits.

Accuracy, speed and checking answers. In all exercises accuracy should be insisted upon. It is not desirable to urge accuracy at the sacrifice of speed.[45] Rather, accuracy should be secured by means of checking answers and by placing emphasis upon speed. The following checks[46] may be introduced in the fourth grade.

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[42]Drushel, J. Andrew, Noonan, Margaret E., and Withers, John W. Arithmetic Essentials—Book One. Chicago: Lyons and Carnahan, 1921, p. 161.

[43]Twentieth Yearbook of the National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Company, 1921, p. 39.

[44]*Ibid.*, p. 57.

[45]Monroe, Walter S. "Principles of method in teaching arithmetic, as derived from scientific investigation." Eighteenth Yearbook of the National Society for the Study of Education, Part II. Bloomington, Illinois: Public School Publishing Company, 1919, p. 89.

[46]Quoted from:
"Arithmetic—course of study for grades four, five, and six." Baltimore, Maryland: Board of Education, 1924, p. 30-32.

1. Approximating results:

Approximation of results, *i.e.*, estimating the results through a reasonable comparison of the facts given and required, is a most valuable check in the solution of problems, immediately showing the absurdity of a grossly incorrect answer.

For example, in the problem, "One book cost $.75. Find the cost of 12 books," pupils should be taught to approximate the answer as follows:

12 books at $1.00 each will cost $12.00. $.75 is less than one dollar, therefore, 12 books at $.75 will cost less than $12.00.

This approximation would immediately show the absurdity of such a result as $900.00, which was the answer actually given by a pupil with no appreciation of the fact that the answer was not reasonable.

2. Check for addition:

The best check for addition is to add each column from the bottom up, writing the sum at the right of the column; then to add the same column from the top down, checking the sum if correct. Write the right hand digit of the sum beneath the column added and add the other digit to the next column.

624	27 ✓
239	18 ✓
876	22 ✓
548	
2287	

Reference: Bailey, Teaching Arithmetic, p. 68.

3. Check for subtraction:

To check subtraction, add the subtrahend and remainder. The result should be the minuend.

179807
94926
84881

Without re-writing any figures, have pupils add the subtrahend and the remainder as follows:

6 and 1 are 7, 2 and 8 are 10, carry 1; 10 and 8 are 18; carry 1; 5 and 4 are 9; 9 and 8 are 17.

4. To check multiplication:

a. Multiply the multiplier by the multiplicand.

Original example	*Check*
728	35
35	728
3640	280
2184	70
25480	245
	25480

b. Divide the product by either of the factors; the result should be the other factor.

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   728
35)25480
   245
    98
    70
   280
   280
```

5. Check for division:

Multiply the quotient by the divisor. If there is a remainder add it to the product. The result should equal the dividend.

Original example	*Check*
24	24
312)7614	312
624	48
1374	24
1248	72
126	7488
	126
	7614

NOTE—The quotient should not be used as a multiplier as errors occurring in the original example may be repeated in the check.

Short cuts. Speed may be furthered by the use of short cuts that are real time-saving devices. The following short methods should be taught in the fourth grade.[47]

1. Addition:

Addition of 2 numbers of 2 orders may be taught by adding, first, the tens digits and then, the units digits of the addends.

For example: To find the sum of 38 and 43.
Think 38 + 40 + 3. Say 38, 78, 81.

2. Subtraction

To subtract 43 from 81, reverse the process used for addition.
Think 81 minus 40 = 41; 41 minus 3 = 38.
Say 81, 41, 38.

Reference: KLAPPER, The Teaching of Arithmetic, p. 166, 173.

3. Multiplication by 10, 100, and 1000.

Children should be taught to use the short method of multiplying by powers of 10, *i.e.*, to multiply an integer by 10, annex one zero; to multiply by 100, annex two zeros; to multiply by 1000, annex three zeros.

38	45	76
10	100	1000
380	4500	76000

Reference: WENTWORTH-SMITH, School Arithmetics. Book One, p. 132.

4. Division by 10, 100, and 1000.

To divide an integer by 10, 100, or 1000 cut off from the right of the dividend as many figures as there are zeros in the divisor.

The quotient is the number expressed by the figures at the left of the line. The remainder is the number expressed by the figures at the right.

10)380	100)4579	1000)58371
	$45\frac{79}{100}$	$58\frac{371}{1000}$
$1\|0)38\|0$	$1\|00)45\|79$	$1\|000)58\|371$

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[47] Quoted from:
"Arithmetic—course of study for grades four, five, and six." Baltimore, Maryland: Board of Education, 1924, p. 32-33.

2. Methods of Instruction

Motivation. The exercises in Drushel, Noonan, and Witmers' book have in themselves a very large motivating element, that is, they interest the pupils because of their nature. For instance, there are colored pictures, other children are represented as doing things (things that children like to do), there is a great deal of variety in the kinds of things about which problems are made, there are things suggested which pupils can make at school or at home, and there are other similar motivating elements. But the teacher should not forget that there are many bases of motivation to which she may appeal and of which she should take advantage. Among those that are especially good in the fourth grade are physical activity, desire to construct things, personal rivalry, group rivalry, the "I'll show you" attitude toward trying tasks, interest in the familiar, interest in the novel, imitation, desire for excellence, self-esteem, and curiosity. Of these, the four, the "I'll show you" spirit, group rivalry, desire for excellence, and self-esteem, should be given particular attention because, at about this time in the pupils' lives, such attitudes are developing rapidly and should be directed toward desirable ends.

Games. As noted in the preceding paragraph, the learning exercises, particularly the problems, have a large motivating element in themselves. There are less inherent motivating elements in the drill exercises. Because of this, the teacher will need to give more attention to motivation in drill exercises than in problems. Much of this can be done through games. The teacher should devise or select suitable games,[48] but she must take care that the games are effective as learning exercises and not merely entertaining or distracting. Lennes[49] lists the following characteristics that should be possessed by a good number game:

a. It should keep all the children at work during practically the whole period devoted to the game.

b. It should not lead to noise or confusion which distracts the attention from the subject.

c. It should create in the child a keen desire to learn.

d. It should make the element of group rivalry stand out sharply.

[48] Some of the best suggestions are to be found in educational periodicals and in books on the teaching of arithmetic.

[49] Lennes, N. J. The Teaching of Arithmetic. New York: The Macmillan Company, 1923, p. 141-45.

e. It should make the group exert pressure on the individual to do his best work.

f. It should make each pupil vitally interested in the work of the other members of his group.

The following example of a good game is given: A class is divided into opposing teams, and perhaps given the names of two colleges or of two athletic teams in which all the pupils are interested. All are provided with pencils and paper. As the teacher shows cards, each of which contains two numbers to be added (they could just as well be for multiplication, subtraction, or division), the pupils write down the sum. After a certain number of cards, for instance ten, have been shown, the pupils trade papers so that each has a paper for some member of the opposing team. Then the teacher reads the correct answers in the order in which they were given, and the number of mistakes for each team is quickly found. This is a very simple game, but it possesses all the good elements enumerated on page 32, especially if it is played from time to time with the same teams and a record is kept of the scores. The game may be varied in several ways, such as, by having the boys and girls oppose each other or by selecting captains to choose teams.

The following game is given by Lennes as an example of a bad game: One pupil is "it." He stands before the class and says, "I am thinking of two numbers whose sum is fifteen," or any other number he may have in mind. The other children then ask in turns, "Is it seven and eight are fifteen?" "Is it ten and five?" "Is it nine and six?" and so on until someone guesses the combination of which the first pupil is thinking. The one who first guesses right is then "it" and the process is started all over again. The bad elements in this game are apparent. The pupils cannot possibly all be kept busy any considerable part of the time, for as soon as a pupil has thought of a combination, all he has to do is to keep it in mind until his turn comes or until someone else gives the same combination. There is likely to be little real rivalry, for the element of chance is too prominent. Furthermore the rivalry that exists is individual, and the game could scarcely be so modified as to bring about group rivalry.

Evaluation of these two games by means of the criteria for judging a number game, page 32, will easily bring out other points of excellence and of weakness. These criteria should be used by teachers whenever their pupils play games, for the harm resulting

from a poor game may be as great as the good that could be accomplished by a well-chosen game.

Oral and written work. Little needs to be said here about the proportion of oral to written work, for adequate provision is made in the textbook. When omissions are made from the text, care should be taken not to underestimate the value of oral work. Rapid written work on problems does not imply corresponding speed and accuracy on oral ones. Many oral exercises and problems are needed if skill is expected in purely mental operations. Furthermore, oral problems (simple ones, it is true) are assuming a larger and larger place in everyday life, so that training in them is appropriate not only for obtaining facility in mental processes but also for meeting life situations.

Proper use of the textbook. As has been indicated throughout the preceding pages, the textbook is the chief source of subject-matter and learning exercises, although it is not to be followed slavishly. Several places have been pointed out as in need of supplementing. Each teacher will find other places that need supplementing for her class as well as some that may be omitted because of the special aptitudes of her pupils. Most of these minor variations must be left to the individual teacher. She must remember that the textbook is the most economical source of subject-matter and of learning exercises which she has available and that variations from it are justifiable only in the degree to which they aid in achieving more efficiently the objectives of the course in arithmetic.

Use of practice tests. If the Courtis Standard Practice Tests are employed, they should be used systematically and according to the accompanying instructions. Even though a teacher may have used the tests before, she should read again the instructions and suggestions; for often she will discover that she has previously overlooked ways in which she could use the tests more effectively and thus secure better results. These tests furnish only a limited type of training and test only particular abilities. Other phases of arithmetic, such as problem solving, must not be neglected.

Provisions for individual differences. The use of practice tests enables the teacher to make some provisions for individual differences, such as excusing from further drill those pupils who have achieved the standard attainment. Other provisions for individual differences are made in the statement of specifications, pages 23-26.

A few objectives and topics are starred because they are intended only for the brighter pupils, and extended time limits are set for the slower pupils on some of the standards of attainment. But there are several provisions for individual differences which more properly come under method, if the term method is extended to include learning exercises and management. For example, there may be no need for the brighter pupils to do the practice exercises on pages 192-93; for if they have attained the standard abilities in the processes involved by the time these exercises are reached, the extra practice would be undesirable.[50] For some of the slower pupils, it may be necessary to devise additional exercises in order that they may attain the desired ability. The brighter children may often be set at acquiring skill after having a principle explained to them while the slower ones are retained for work preliminary to starting a principle. Thus, in learning to multiply or divide by multiples of ten, the general principles of adding zeros to the multiplicand or of cutting off a corresponding number of places from the dividend may be grasped immediately by some pupils, while others will need to do a number of preliminary exercises before they grasp the general principle underlying the processes.

Especial attention needs to be given to discovering and appealing to the interests of the slower pupils in order that their interest in arithmetic may be aroused and maintained. Ordinarily the bright pupils like whatever they are asked to do, but they are in danger of being bored when the tasks are too easy or when there is too much repetition.

The biggest question that arises in providing for individual differences is more often one of management than of method or curriculum. While the slower pupils are still working, what is to be done with those who have completed the task? Small adjustments in assignments of written work easily care for minor variations, but large differences must be provided for in some other way. If pupils are excused from work of the type that most of the class are on, they may take up some other subject, but ordinarily they have as little need of additional work in other subjects as in arithmetic. An opportunity to enrich their arithmetical concepts is offered to the teacher. Projects such as those suggested for additional learning exercises on

[50]When pupils are excused from further practice on a given process, they should be tested at intervals in order to make sure that they are maintaining the standard ability.

page 29 allow the brighter pupils to exercise and develop further the initiative they already possess.

All provisions for individual differences should be based upon definitely established facts in so far as possible. This means that there must be constant testing. Fortunately, the textbook in this grade provides a number of tests on the fundamental operations which may be used; but there are not enough tests, such as those on page 201, distributed throughout the book. After a process has been studied for a short time, tests should be given in order to see what progress has been made, to check the effectiveness of the teaching, and to reveal difficulties to both teacher and pupil. It is suggested that the teacher devise and give brief surprise tests[51] (not over ten minutes) when the following pages are reached: p. 161, 165, 174, 179, 184, 189, 193, 197, 201, 206, 212, 213, 215, 218, 223, 224, 229, 234, 239, 253, 265, 278, 281, and 304.

Use of standardized tests. Although it is possible for all testing to be done by means of tests constructed by the teacher, standardized tests offer the advantages of comparative scores, thoroughness of diagnosis, and so forth, which the former do not have. However, the justification for the administration of standardized tests depends upon the uses made of the results. If such tests are decided upon, they should be used systematically and in accordance with the following suggestions:

Early in the year, preferably as soon as the class is well started, the pupils are familiar with the teacher, and their acquaintance with arithmetic is renewed, Monroe's General Survey Scales in Arithmetic, Scale I, either form, should be given.

For purposes of diagnosis, Monroe's Diagnostic Tests in Arithmetic, Parts I and II, and Monroe's Standardized Reasoning Tests in Arithmetic should be administered near the end of the first semester. Remedial work based upon the results should be given.[52]

As an aid in determining promotion, Monroe's General Survey Scales in Arithmetic, Scale I, a form different from that used at the beginning of the year, should be given near the close. The pupils should achieve the October norm for the fifth grade. The other objectives set forth on pages 23-26 should also be achieved.

[51]For a good discussion of the general principles of testing, see:
Monroe, Walter Scott, DeVoss, James Clarence, and Kelly, Frederick James. Educational Tests and Measurements. Boston: Houghton Mifflin Company, 1924, p. 469-86.

[52]For a discussion of these tests and the use to make of the results, see:
Ibid., p. 41-49, 58-64, and 68-89.

STANDARDS OF ATTAINMENT IN THE FUNDAMENTAL PROCESSES[53]

The following standards for each grade are taken from the field of standardized tests, namely, the Monroe Diagnostic Tests in Arithmetic and the Courtis Standard Research Tests, Series B. They indicate what the average child should be able to accomplish at the end of each grade. For example, a child of the fourth grade is supposed to perform correctly a certain number of examples similar to the ones illustrated for his grade within the time limits specified. These standards or objectives are definite. There is no doubt in the mind of the teacher as to what her pupils of average intelligence should be able to accomplish. At present, no definite standards have been determined for Grades I, II, and III, but those for the remaining grades of the elementary school are given.

Fourth-grade standards. A fourth-grade child should perform correctly the following types of examples at the following rates:

A. Addition.

Example	*Number*	*Time*
4 7 2	16	1 min.
7 6 6 5 0 5 1 8 7 3 3 1 2	5	4 min.
927 379 756 837 924 110 854 965 344	6	8 min.

[53] Ruth Streitz, formerly Associate, Bureau of Educational Research, should be given the credit for bringing these standards together.

B. Subtraction

Example	*Number*	*Time*
37 5	7	1 min.
739 367	4	1 min.
107795491 77197029	7	4 min.

C. Multiplication

Example	*Number*	*Time*
6572 6	2	1 min.
8246 29	1	3 min.

D. Division

Example	*Number*	*Time*
8)3840	1	1 min.
25)6775	2	4 min.

Fifth-grade standards. A fifth-grade pupil should perform correctly the following types of examples, at the following rates:

A. Addition

Example	*Number*	*Time*
2 6 7	24	1 min.
6 8 8 9 9 5 1 8 7 7 4 5 4	6	4 min.
297 925 473 983 315 661 794 177 124	8	8 min.

B. Subtraction

Example	*Number*	*Time*
41 8	14	1 min.
508 447	6	1 min.
75088824 57406394	9	4 min.

C. Multiplication

Example	*Number*	*Time*
5862 2	4	1 min.
560 37	4	4 min.
6942 58	5	6 min.

D. Division

Example	*Number*	*Time*
$4\overline{)7432}$	2	1 min.
$43\overline{)1591}$	3	4 min.
$94\overline{)853252}$	6	8 min.

E. Fractions

Example	*Number*	*Time*
$1/6 + 1/3$	4	3 min.
$3/4 - 2/5$	3	4 min.
$2/3 \times 3/4$	7	2 min.
$2/5 \div 1/3$	5	4 min.

Sixth-grade standards. A sixth-grade pupil should perform correctly the following types of examples at the following rates:

A. Addition

Example	*Number*	*Time*
8 0 5	26	1 min.
6 3 6 4 8 9 2 0 1 2 1 2 4	8	4 min.

Example	*Number*	*Time*
136 340 988 386 353 904 547 192 439	10	8 min.

B. Subtraction

Example	*Number*	*Time*
53 − 9	18	1 min.
962 − 325	8	1 min.
91050005 − 19901563	11	4 min.

C. Multiplication

Example	*Number*	*Time*
2845 × 8	5	1 min.
690 × 70	8	4 min.
5379 × 85	8	6 min.

D. Division

Example	*Number*	*Time*
$6\overline{)4680}$	6	2 min.
$37\overline{)9990}$	8	8 min.

E. Common fractions

Example	*Number*	*Time*
$3/10 + 2/5$	5	3 min.
$5/6 - 3/4$	3	4 min.
$2/5 \times 3/7$	5	1 min.
$4/7 \div 2/3$	6	4 min.

F. Decimal fractions

Multiplication: The decimal point is to be placed correctly in the products already given.

Example			*Number*	*Time*
657.2 × .7 = 46400	82.74 × .4 = 33096	5.863 × .6 = 35178	24	1 min.
932.7 × .08 = 74616	67.50 × .03 = 20250	8.409 × .07 = 58863		

Example				Number	Time
4065.	967.5	14.53	8.637		
5.1	8.4	6.2	1.6		
207315	712700	90086	138192		
				25	1 min.
7486.	907.2	61.32	2.893		
.76	.39	.17	.68		
558936	353808	104244	196724		

Division: The answers are given without the decimal point. Each answer is to be written in its proper position and the decimal point inserted in its proper place.

Example			Number	Time
.4)148	Ans.:	37		
.9)65.7	Ans.:	73		
.2)7.92	Ans.:	396	5	1 min.
.7).301	Ans.:	43		
.03)16.2	Ans.:	54		
.06)7.44	Ans.:	124	3	1 min.
.02).144	Ans.:	72		
.43)1591.	Ans.:	37		
.63)35.91	Ans.:	57		
2.1)140.7	Ans.:	67		
			7	2 min.
2.8)21.980	Ans.:	785		
83.)531.2	Ans.:	64		
79.)36.893	Ans.:	467		

Seventh-grade standards. A seventh-grade pupil should perform correctly the following types of examples at the following rates:

A. Addition

Example	Number	Time
486		
765		
524		
140		
812	11	8 min.
466		
355		
834		
567		

B. Subtraction

Example	Number	Time
87939983	12	4
72207316		

C. Multiplication

Example	*Number*	*Time*
2648 46	10	6 min.

D. Division

Example	*Number*	*Time*
86)80066	4	8 min.

E. Common fractions

Example	*Number*	*Time*
$4/5 + 7/10$	7	3 min.
$8/15 - 4/9$	5	4 min.
$4/5 \times 7/9$	13	2 min.
$4/5 \div 1\frac{1}{2}$	10	4 min.

F. Decimal fractions

Multiplication: The decimal point is to be placed correctly in the products already given.

Example				*Number*	*Time*
657.2 .7 46400	82.74 .4 33096	5.863 .6 35178		23	1 min.
932.7 .08 74616	67.50 .03 20250	8.409 .07 58863			
4065. 5.1 207315	967.5 8.4 712700	14.53 6.2 90086	8.637 1.6 138192	22	1 min.
7486. .76 558936	907.2 .39 353808	61.32 .17 104244	2.893 .68 196724		

Division: The answers are given without the decimal point. Each answer is to be written in its proper position and the decimal point inserted in its proper place.

Example			*Number*	*Time*
.4)148	Ans.:	37	4	1 min.
.9)65.7	Ans.:	73		
.2)7.92	Ans.:	396		
.7).301	Ans.:	43		

Example			*Number*	*Time*
.03)16.2	Ans.:	54		
.06)7.44	Ans.:	124	3	1 min.
.02).144	Ans.:	72		
.43)1591.	Ans.:	37		
.63)35.91	Ans.:	57		
2.1)140.7	Ans.:	67		
2.8)21.980	Ans.:	785	4	1 min.
83.)531.2	Ans.:	64		
79.)36.893	Ans.:	467		

Eighth-grade standards. An eighth-grade pupil should perform correctly the following types of examples at the following rates:

A. Addition

Example	*Number*	*Time*
176 783 697 200 366 851 535 323 229	12	8 min.

B. Subtraction

Example	*Number*	*Time*
160670971 80361837	13	4 min.

C. Multiplication

Example	*Number*	*Time*
4263 37	11	6 min.

D. Division

Example	*Number*	*Time*
73)58765	5	8 min.

E. Common fractions

Example	*Number*	*Time*
$5/8 + 3/4$	10	3 min.
$4/5 - 1/3$	7	4 min.
$1/6 \times 3/10$	15	2 min.
$5/12 \div 4/9$	7	2 min.

F. Decimal fractions

Multiplication: The decimal point is to be placed correctly in the products already given.

Example				*Number*	*Time*
657.2	82.74	5.863			
.7	.4	.6			
46400	33096	35178			
				26	1 min.
932.7	67.50	8.409			
.08	.03	.07			
74616	20250	58863			
4065.	967.5	14.53	8.637		
5.1	8.4	6.2	1.6		
207315	712700	90086	138192		
				27	1 min.
7486.	907.2	61.32	2.893		
.76	.39	.17	.68		
558936	353808	104244	196724		

Division: The answers are given without the decimal point. Each answer is to be written in its proper position and the decimal point inserted in its proper place.

Example			*Number*	*Time*
.4)148	Ans.:	37		
.9)65.7	Ans.:	73		
.2)7.92	Ans.:	396	7	1 min.
.7).301	Ans.:	43		
.03)16.2	Ans.:	54		
.06)7.44	Ans.:	124	5	1 min.
.02).144	Ans.:	72		
.43)$\overline{1591.}$	Ans.:	37		
.63)$\overline{35.91}$	Ans.:	57		
2.1)$\overline{140.7}$	Ans.:	67	5	1 min
2.8)$\overline{21.980}$	Ans.:	785		
83.)$\overline{531.2}$	Ans.:	64		
79.)$\overline{36.893}$	Ans.:	467		

SELECTED AND ANNOTATED BIBLIOGRAPHY

Introductory statement. This bibliography is made up of references that will be helpful to those who make courses of study in arithmetic. No attempt has been made to include all possible references. The bibliography is divided into five groups: first, general references on curriculum and course-of-study making; second, books and articles on methods of teaching arithmetic; third, arithmetic courses of study; fourth, references on testing and standards of achievement; and fifth, reports of investigations and miscellaneous references.

I. GENERAL REFERENCES ON CURRICULUM AND COURSE-OF-STUDY MAKING

Bobbitt, Franklin. How to Make a Curriculum. Boston: Houghton Mifflin Company, 1924, p. 1-75.

This is a report of the work on revising the curriculum in Los Angeles, which Dr. Bobbitt directed over a period of two years.

Caldwell, Otis W. "Types and principles of curricular development." Teachers College Record, 24:326-37, September, 1923.

Speech delivered at meeting of the Department of Superintendence of the National Education Association at Cleveland, February 28, 1923. Outlines the method and results of two types of curricular investigations and states certain principles for use in reorganizing school subjects of study.

Charters, W. W. Curriculum Construction. New York: The Macmillan Company, 1923, p. 3-168.

This portion of the book gives a good background theory of curriculum construction and presents Dr. Charters' own point of view.

McMurry, Charles A. How to Organize the Curriculum. New York: The Macmillan Company, 1923. 358 p.

The curriculum is discussed in terms of projects, type studies, and large units of study. A suggested curriculum of large teaching units is given, covering the fields of geography, history, science, and literature.

Monroe, Walter S. "Making a course of study." University of Illinois Bulletin, Vol. 23, No. 2, Bureau of Educational Research Circular No. 35. Urbana: University of Illinois, 1925. 35 p.

This circular presents the best present day ideas on the general make-up of courses of study, the way to go about making a course of study, and the benefits to be derived from such work. A lengthy bibliography on curriculum and course-of-study making is included.

Threlkeld, A. L. "Curriculum revision: how a particular city may attack the problem," Elementary School Journal, 25:573-82, April, 1925.

This is a report of the method of attack used in Denver, Colorado.

Wilson, H. B. "The course of study in the work of the modern school," Course of Study Monographs, Introductory. Berkeley, California: Board of Education, 1921. 14 p.

"Introductory to all (Berkeley) Courses of Study presenting the general point of view which has guided the formulation of the detailed course in all subjects for the various schools." (Introductory Note.)

"The elementary school curriculum." Second Yearbook of the Department of Superintendence, Washington: Department of Superintendence of the National Education Association, 1924. 296 p.

A fair presentation of the elementary curriculum situation in the United States in 1923 is given.

"Facts on the public school curriculum." Research Bulletin of the National Education Association, Vol. I, No. 5. Washington: Research Division of the National Education Association, 1923, p. 310-50.

This bulletin furnishes good source material on time allotments, statutory requirements, grade combinations of subjects, and other pertinent matters.

II. METHODS OF TEACHING ARITHMETIC

Brown, Joseph C. and Coffman, Lotus D. How to Teach Arithmetic. Chicago: Row, Peterson and Company, 1914. 373 p.

Although more than ten years old, this book is not at all out of date either in point of view or in helpful suggestions.

Charters, W. W. Teaching the Common Branches. Boston: Houghton Mifflin Company, 1913, p. 278-99.

The pages selected contain a particularly valuable discussion on the use of textbooks.

Klapper, P. The Teaching of Arithmetic. New York: D. Appleton and Company, 1921. 393 p.

One of the best books on methods of teaching arithmetic.

Lennes, N. J. The Teaching of Arithmetic. New York: The Macmillan Company, 1923. 486 p.

A stimulating book combining theory with illustrations.

Monroe, Walter S. "Principles of method in teaching arithmetic, as derived from scientific investigations," Eighteenth Yearbook of the National Society for the Study of Education, Part II. Bloomington, Illinois: Public School Publishing Company, 1919, p. 78-95.

Twenty-five fundamental principles of method are stated and briefly discussed.

Osburn, Worth J. Corrective Arithmetic. Boston: Houghton Mifflin Company, 1924. 182 p.

A rather comprehensive treatment of pupils' difficulties and ways of meeting them.

Streitz, Ruth. "Teachers' difficulties in arithmetic and their correctives." University of Illinois Bulletin, Vol. 21, No. 34, Bureau of Educational Research Bulletin No. 18. Urbana: University of Illinois, 1924. 34 p.

Twenty-eight difficulties are listed with correctives that are actually in successful use.

Thorndike, Edward Lee. The New Methods in Arithmetic. Chicago: Rand McNally and Company, 1921. 260 p.

Abundant detailed illustrations and applications are given for the principles discussed.

III. COURSES OF STUDY IN ARITHMETIC

"Arithmetic." Course of Study Monograph, Elementary Schools, No. 1. Berkeley, California: Board of Education, 1921. 91 p.

Course of study for the first six grades, divided into semesters. Especially good on methods and learning exercises.

"Arithmetic—course of study for grades four, five, and six." Baltimore, Maryland: Department of Education, 1924. 111 p.

Especially helpful in outlining of subject-matter and in suggestions on methods of teaching.

"Arithmetic—elementary course of study." Trenton, New Jersey: Board of Education, 1923. 96 p.

Some good suggestions on provisions for individual differences and on general objectives.

"Arithmetic—grades 1, 2, 3, 4, 5, and 6—course of study monograph." Denver: Board of Education, 1924. 228 p.

Probably the most comprehensive course of study in arithmetic yet published. Very suggestive.

"Arithmetic—syllabus for elementary schools." University of the State of New York Bulletin, No. 815. Albany: University of the State of New York Press, 1925. 121 p.

This is not a course of study but is only a syllabus, containing both condensed and expanded outlines for the eight elementary grades. Many helpful suggestions on methods of teaching are given throughout the expanded outlines.

"Course of study, public schools, Baltimore County, Maryland, Grades I-VIII." Baltimore, Maryland: Warwick and York, 1921, p. 261-329.

The parts for the first four grades are the best. Most helpful in the matter of time allotments within recitations and games.

"Geography—history—arithmetic—course of study for kindergarten and grades one, two and three." Baltimore, Maryland: Department of Education, 1924. 78 p.

This contains the portion of the course of study in arithmetic which precedes that given previously in this bibliography.

"Long Beach City Schools course of study." Long Beach, California: Board of Education, 1924. (Five separate monographs for the first six grades, having from 19 to 40 pages each.)

These course-of-study monographs are especially good in making provisions for individual differences and in showing what the course of study can do by way of directing teachers in the use of the textbook.

IV. TESTING AND STANDARDS OF ACHIEVEMENT

Doherty, Margaret and MacLatchy, Josephine. "Bibliography of educational and psychological tests and measurements." United States Bureau of Education Bulletin, 1923, No. 55. Washington: Government Printing Office, 1924. 233 p.

This bibliography gives not only the tests but a rather complete list of references that discuss the particular tests, the uses of tests in general, and the uses of tests according to types of schools.

Monroe, Walter Scott, DeVoss, James Clarence, and Kelly, Frederick James. Educational Tests and Measurements. (Revised Edition.) Boston: Houghton Mifflin Company, 1924, p. 1-93, 417-30, 469-86.

The structure, uses, and limitations of most of the standardized tests in arithmetic are discussed in the sections referred to. The general theory of testing is discussed and practical suggestions made. An excellent bibliography on testing in arithmetic is given on pages 91-93.

Odell, Charles W. "Educational tests for use in elementary schools, revised." University of Illinois Bulletin, Vol. 22, No. 16. Bureau of Educational Research Circular No. 33. Urbana: University of Illinois, 1924. 22 p.

An annotated bibliography of tests that are now available. "Tests that are known to be distinctly unsatisfactory are omitted . . ." Norms are available for most of the tests listed. The bibliography is preceded by a brief discussion of the characteristics and use of tests.

Bureau of Cooperative Research (Compiled by). First Revision of Bibliography of Educational Measurements. Bulletin of the School of Education, Vol. 1, No. 5. Bloomington, Indiana: Indiana University, 1925. 147 p.

"This bibliography is compiled for the double purpose of listing all efforts, so far as they have come to our attention, that have been made in the United States to develop achievement tests, and of giving a brief description of each test, including in the description not only an analysis of the test and its purpose but also available information concerning the range of the test, administration cost of the test in the publisher of the test, and the date of publication." (From foreword of first edition.)

V. REPORTS OF INVESTIGATIONS AND MISCELLANEOUS REFERENCES

Buswell, Guy Thomas and Judd, Charles Hubbard. "Summary of educational investigations relating to arithmetic." Supplementary Educational Monographs, No. 27. Chicago: University of Chicago, 1925. 212 p.

A most valuable and usable summary of 307 articles and books of merit which report scientific investigations of the methods and results of teaching arithmetic.

Davis, M. Elsie. "The development of the fundamental number habits," The School Magazine, 4:48-50, October, 1921.

Published by the Board of Education of Buffalo, New York. Contains suggestions on association and learning of the one hundred addition and one hundred subtraction facts.

Davis, Roy. "Business practice in elementary schools." Harvard Bulletins in Education, Vol. 6, Cambridge: Harvard University, October, 1917.

Report on an investigation as to what knowledge of business terms (money, credit, interest, and so forth) is had by children in the elementary school. Contains suggestions on teaching arithmetic with these terms in view.

Jessup, Walter A. "Current practices and standards in arithmetic," Fourteenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1915, p. 116-30.

Makes recommendations, based on judgments and practices of superintendents, as to elimination of topics, increased emphasis, time devoted to recitations, grade occurrence of topics, and objective standards.

Monroe, Walter S. "A preliminary report of an investigation of the economy of time in arithmetic," Sixteenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1917, p. 111-27.

Especially valuable because of the explicit recognition of different types of problems.

Wilson, Guy M. "A survey of the social and business uses of arithmetic," Sixteenth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Company, 1917, p. 128-42.

This is a report of one of the foremost investigations that points out the influence on the arithmetic curriculum of business and social usage.

Wilson, Guy M. "Arithmetic," Third Yearbook of the Department of Superintendence. Washington: Department of Superintendence of the National Education Association, 1925, p. 35-109.

A digest of most of the special studies which have been made on the arithmetic curriculum.

"Material for arithmetical problems." Department of Education, Division of Reference and Research, Bulletin No. 2. New York City: Board of Education, 1914.

This bulletin consists of many arithmetical problems, which are the result of a study made in New York City at the suggestion of S. A. Courtis, one of the members of the School Inquiry Committee. There is also a good discussion of the nature of the arithmetical problem.

On minimum essentials: Fourteenth Yearbook of the National Society for the Study of Education, Part I; Sixteenth Yearbook, Part I; Seventeenth Yearbook, Part I; and Eighteenth Yearbook, Part II. Bloomington, Illinois: Public School Publishing Company, 1915, 1917, 1918, 1919.

These are reports on economy of time and minimal essentials in elementary school subjects. The discussions furnish a background for such provisions in courses of study. Some concrete material and suggestions are also provided.